Globally Optimal Design of Double-Pipe Heat Exchanger Modular Units

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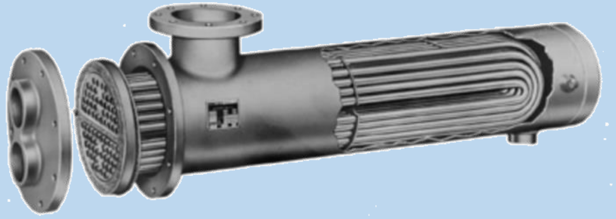
- Most common heat exchanger in the industry

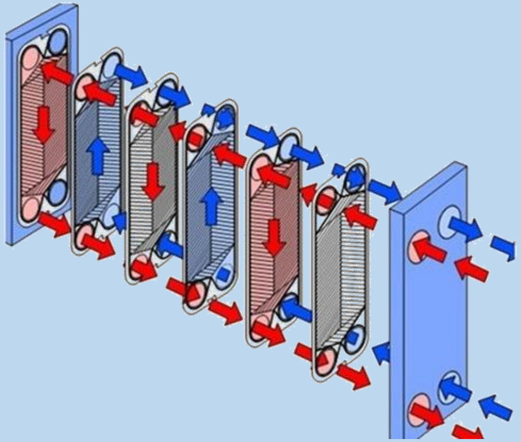
SHELL AND TUBE



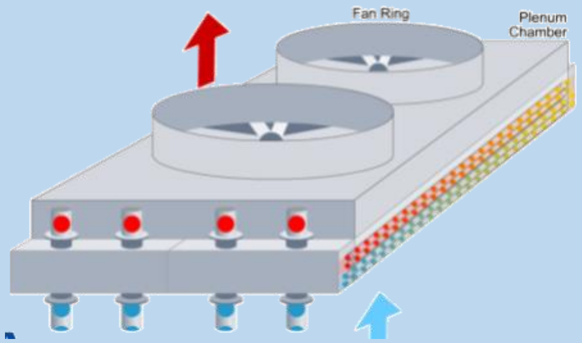
⇨ SCIENTIFIC COMMUNITY LARGEST FOCUS

- Other types of heat exchangers

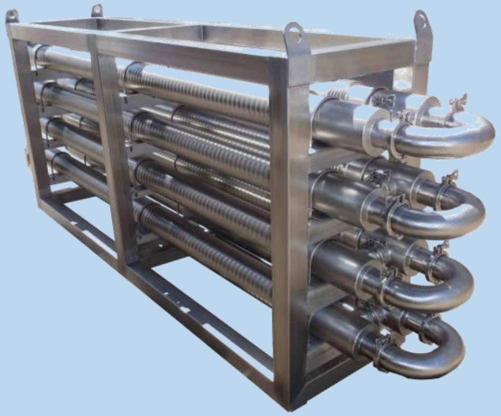
PLATE



AIR COOLERS



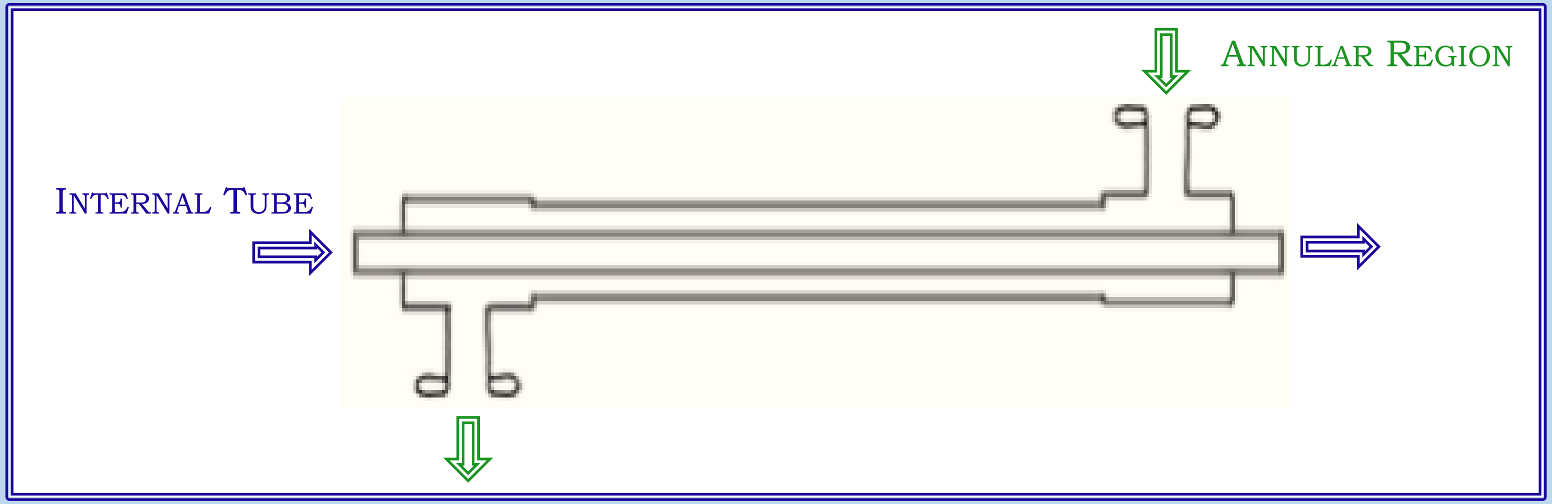
DOUBLE PIPE

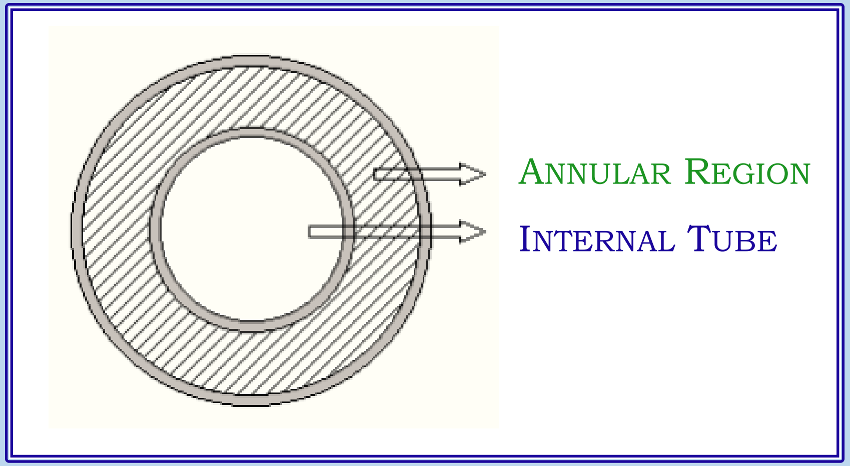


Double Pipe Heat Exchanger in the Chemical Industry

- Services of small magnitude (≤ $50m^2$);
- Large temperature intersection;
- Thermal services involving solids;
- Absence of stagnation regions;
- High pressure services;
- Flexibility to increase or reduce area;
- Multiplicity of operational alternatives;

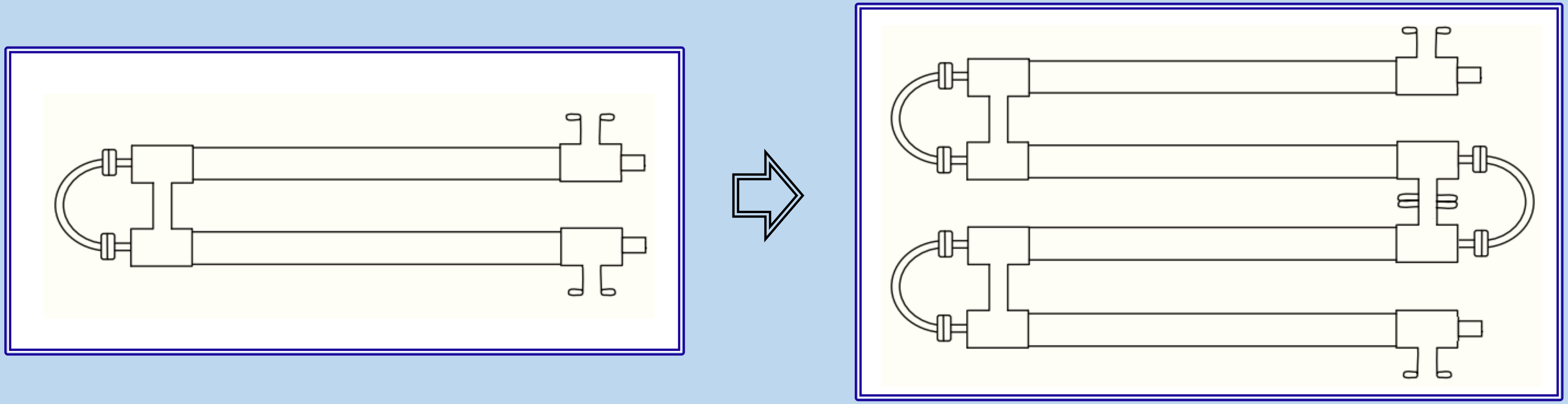
Double Pipe Heat Exchangers – Architecture



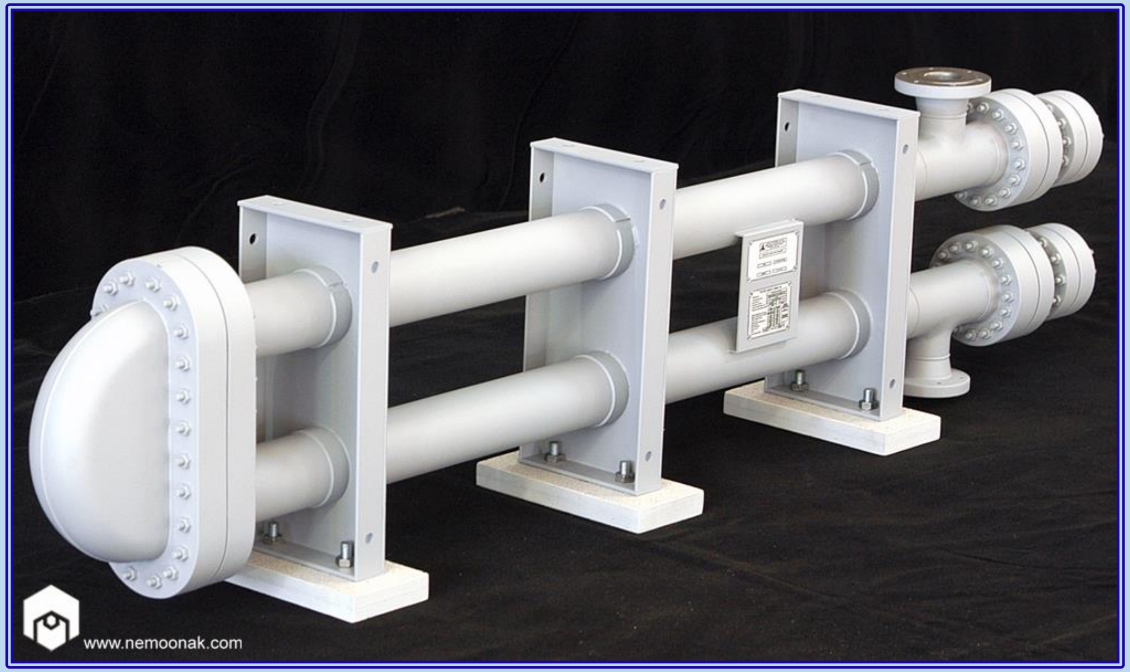


Double Pipe Heat Exchangers – Hairpins

Hairpin Association

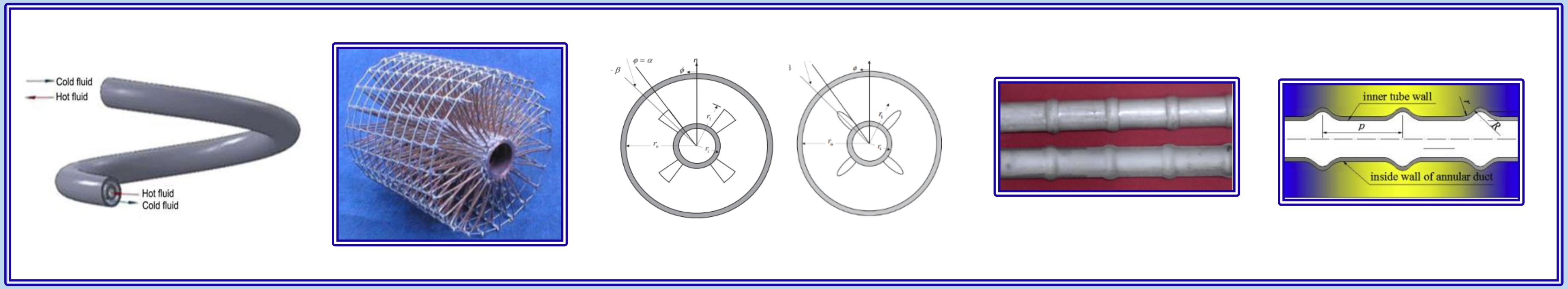


Double Pipe Heat Exchangers – Examples



Double Pipe Heat Exchangers – Literature

- Focus on heat transfer enhancement alternatives:



⇨ Generally know unit dimensions

- Minority of projects aimed at reducing investment costs;
- Simplified approaches and few free design variables.

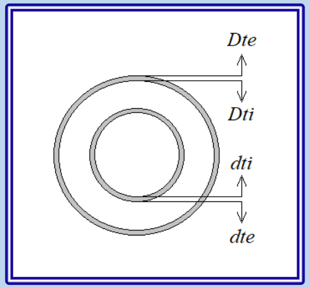
Söylemez (2004):
- Ratio of diameters, length and fluid allocation known;
- Decision variable: Inner tube diameter.

Swamee et al (2008):
- Length and fluid allocation known;
- Decision variables: Pipe diameters.

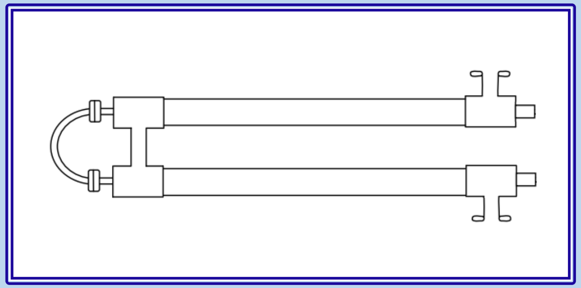
This Work Proposal

- Explore the modular characteristic of double pipe heat exchangers;
- Increase the number of decision variables;

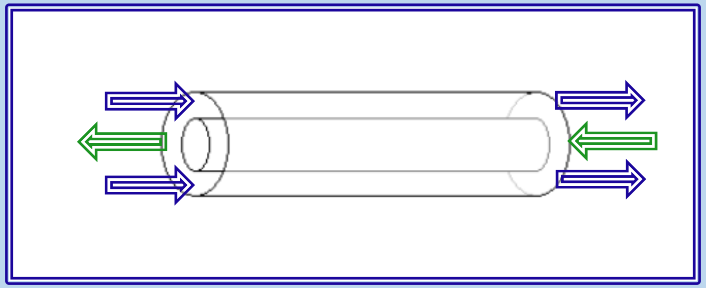
Diameters

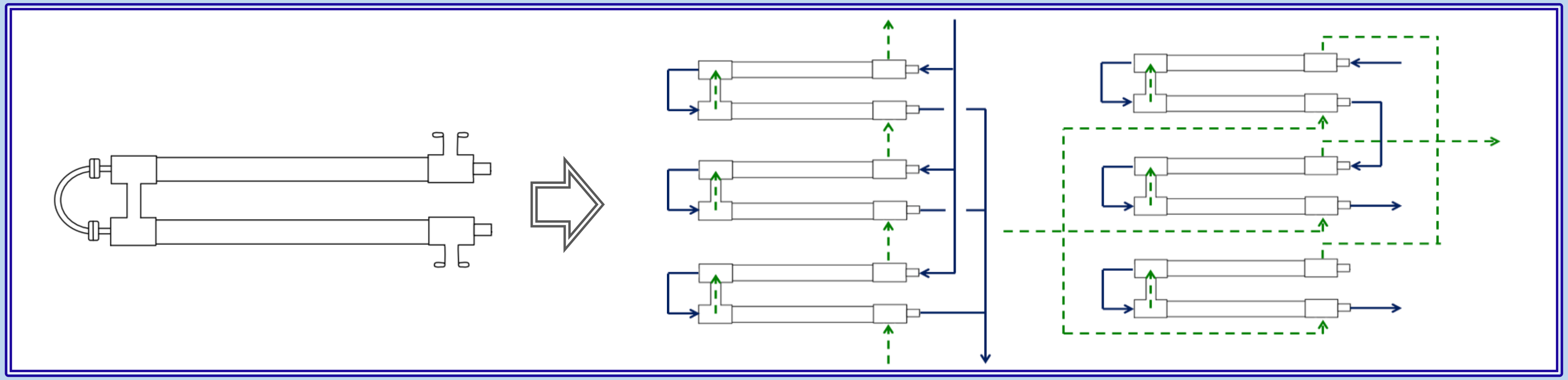


Length



Fluid Allocation



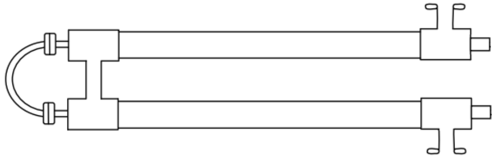


Arrangement

- Different flow regimes: Laminar, Transitional and Turbulent
- Three proposed models: Raw MINLP, Linear-Binary MINLP and MILP

Proposed Architecture

A HAIRPIN

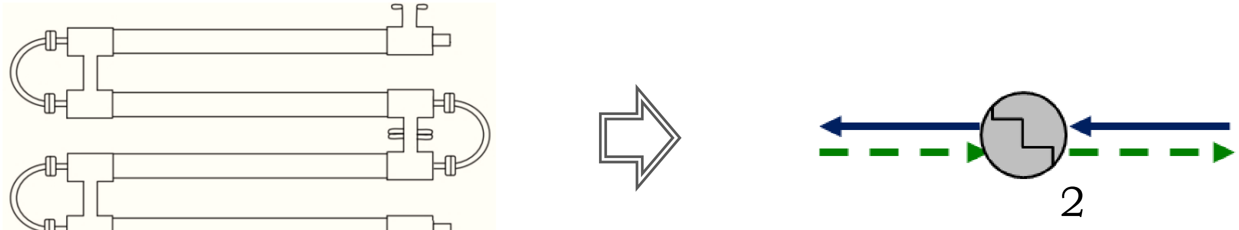


Basic structure. The flow arrangement for a hairpin in this study is always countercurrent

A UNIT

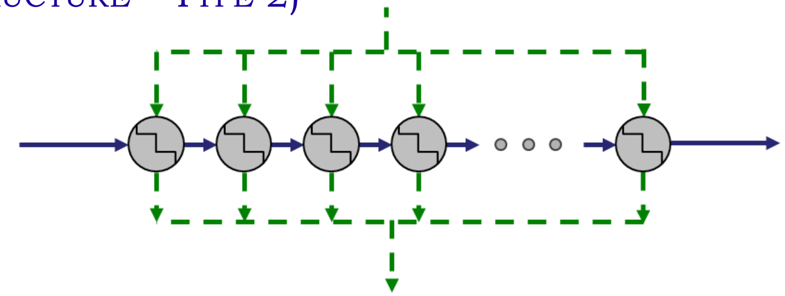
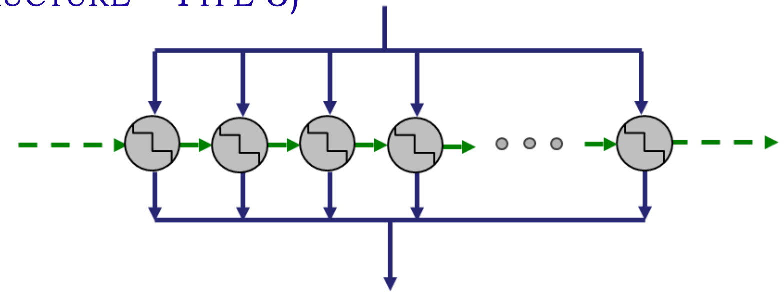
Multiple hairpins connected in series

Example of a unit of two hairpins:



PROPOSED ARCHITECTURE

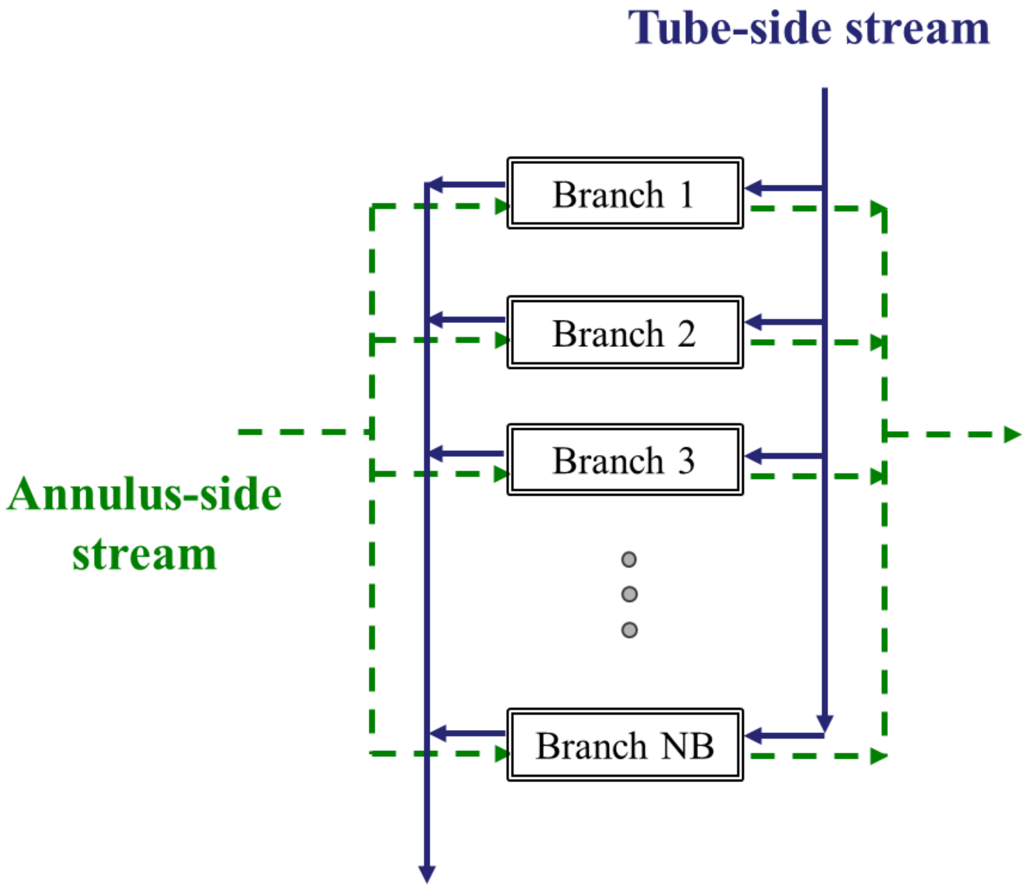
A BRANCH — A structure composed of units that can be arranged in three different ways

	TUBE-SIDE STREAM	ANNULUS-SIDE STREAM
(STRUCTURE - TYPE 1) 	SÉRIES	SÉRIES
(STRUCTURE - TYPE 2) 	SÉRIES	PARALLEL
(STRUCTURE - TYPE 3) 	PARALLEL	SÉRIES

Proposed Architecture

GENERAL STRUCTURE

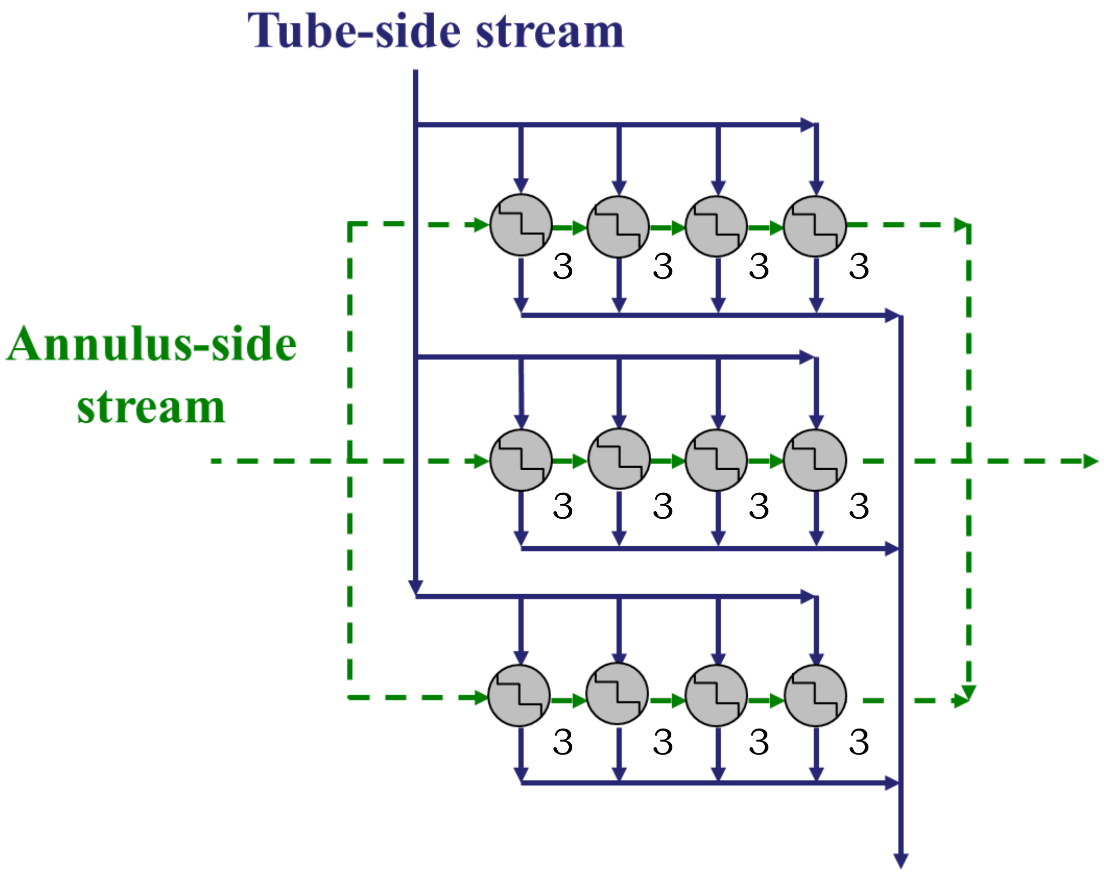
The association of one of the structures in a set of parallel branches



Proposed Architecture

EXAMPLE

Specific example of a General structure with three branches (type III) (four units per branch and 3 hairpins per unit).



General Idea

Decision Variables

Fluid allocation;

Pipe diameters;

Hairpin length;

N° of hairpins/unit;

N° of units in parallel for each flow side/branch;

Number of branches

Constraints

Structural;

Modelling;

Imposed Bounds;

Objective Function

Heat transfer area minimization

Optimization

Software GAMS 23.7

Objective Function

- Heat transfer area minimization

Constraints

- Representation of geometric variables;
- Fluid allocation;
- Structural constraints;
- Thermo hydraulic modeling;
- Heat transfer;
- Velocity and pressure drop bounds.

Geometric Variables Representation

Discrete Options Selection

Diâmetros e comprimento

→ Introdução de variáveis binárias

Parameter	Unit	Discrete Options 1	2	3	4	5	6	7	8	9
NPS	in	½	¾	1	1 ¼	1 ½	2	2 ½	3	3 ½
$\widehat{pdte}$	m	0.021	0.027	0.033	0.042	0.048	0.060	0.073	0.089	0.102

$$\sum_{sd=1}^{sdmax} yd_{sd} = 1 \quad \Rightarrow \quad yd_{sd=3} = 1, \quad yd_{sd\neq 3} = 0$$

$$dte = \sum_{sd=1}^{sdmax} \widehat{pdte}_{sd} yd_{sd} \quad \Rightarrow \quad dte = 0.033\, m$$

Geometric Variables Representation

Structure Selection

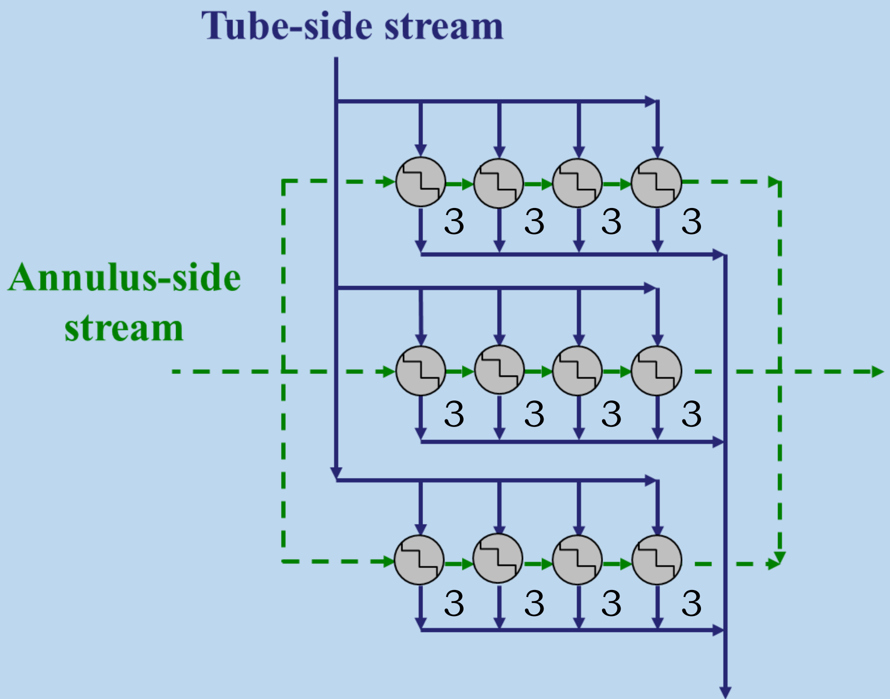
NB – Number of Branches

NPt/NPa – N° of units in parallel per branch for tube and annulus side

Nh – N° of hairpins per unit

$$NB = \sum_{sB=1}^{sBmax} \widehat{pNB}_{sB}\, yB_{sB} \qquad \sum_{sB=1}^{sBmax} yB_{sB} = 1$$

$$NPt = \sum_{sE=1}^{sEmax} \widehat{pNE}_{sE}\, yPt_{sE} \qquad \sum_{sE=1}^{sEmax} yPt_{sE} = 1$$



Three Branches	$NB = 3$
Tube-side in Parallel	$NPt = 4$
Annulus Side in Series	$NPa = 5$
Three Hairpins per Unit	$Nh = 3$

Geometric Variables Representation

Additional Logical Constraints

To ensure that if the tubeside has already more than one parallel passage, the annular side can be only arranged in series and vice versa:

$$yPt_{sE=1} + yPa_{sE=1} \geq 1$$

$$yPa_{sE=1} = 0 \quad \Rightarrow \quad yPt_{sE=1} = 1$$

To force that the outer tube inner diameter is larger than the inner tube outer diameter one writes:

$$\sum_{sD=1}^{sDmax} \widehat{pDti}_{sD} yD_{sD} \geq \sum_{sd=1}^{sdmax} \widehat{pdte}_{sd} yd_{sd} + \varepsilon$$

Fluid Allocation

$$yT_c + yT_h = 1$$

Cold Stram on Tube-Side $yT_c = 1$

$yT_h = 1$ Hot Stream on Tube-Side

Association of streams flows and physical properties with their allocation

$$mt = \widehat{m_c}\, yT_c + \widehat{m_h}\, yT_h \qquad \rho t = \hat{\rho}_c\, yT_c + \hat{\rho}_h\, yT_h$$

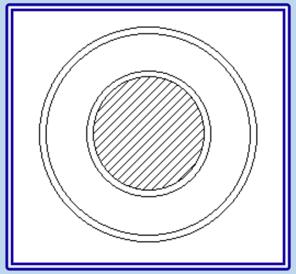
$$ma = \widehat{m_c}\, yT_h + \widehat{m_h}\, yT_c \qquad \rho a = \hat{\rho}_c\, yT_h + \hat{\rho}_h\, yT_c$$

Structural Constraints

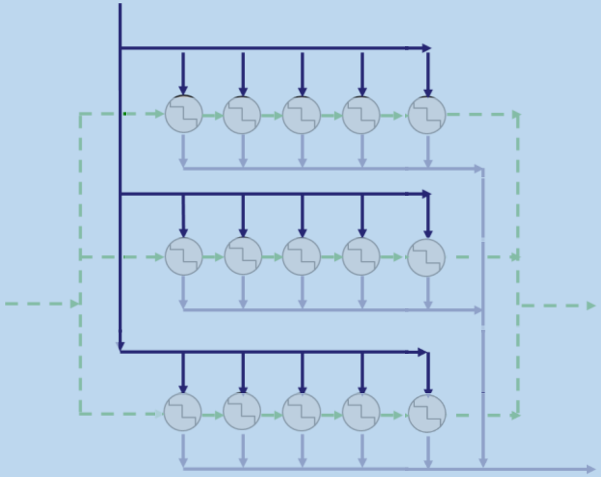
Flow Areas and Flow Path Lengths

Depends on the selected layout

$$At = \left(\frac{\pi dti^2}{4}\right) NB\ NPt$$



$NB = 3 \qquad NPt = 5$



Thermohydraulic Modeling

Dimensionless Numbers and Velocities

$$dh = Dti - dte$$

$$Prt = \frac{Cpt\,\mu t}{kt} \quad Ret = \frac{dti\,vt\,\rho t}{\mu t} \quad vt = \frac{(m\,t/\rho\,t)}{At}$$

$$Pra = \frac{Cpa\,\mu a}{ka} \quad Rea = \frac{dh\,va\,\rho a}{\mu a} \quad va = \frac{(m\,a/\rho\,a)}{Aa}$$

Pressure Drop – Darcy-Weisbach

$$\Delta Pt = \rho t\, ft \frac{Lt}{dti}\frac{vt^2}{2}$$

$$\Delta Pa = \rho a\, fa \frac{Lt}{dh}\frac{va^2}{2}$$

Friction Factors and Nusselt Numbers

Regime switching

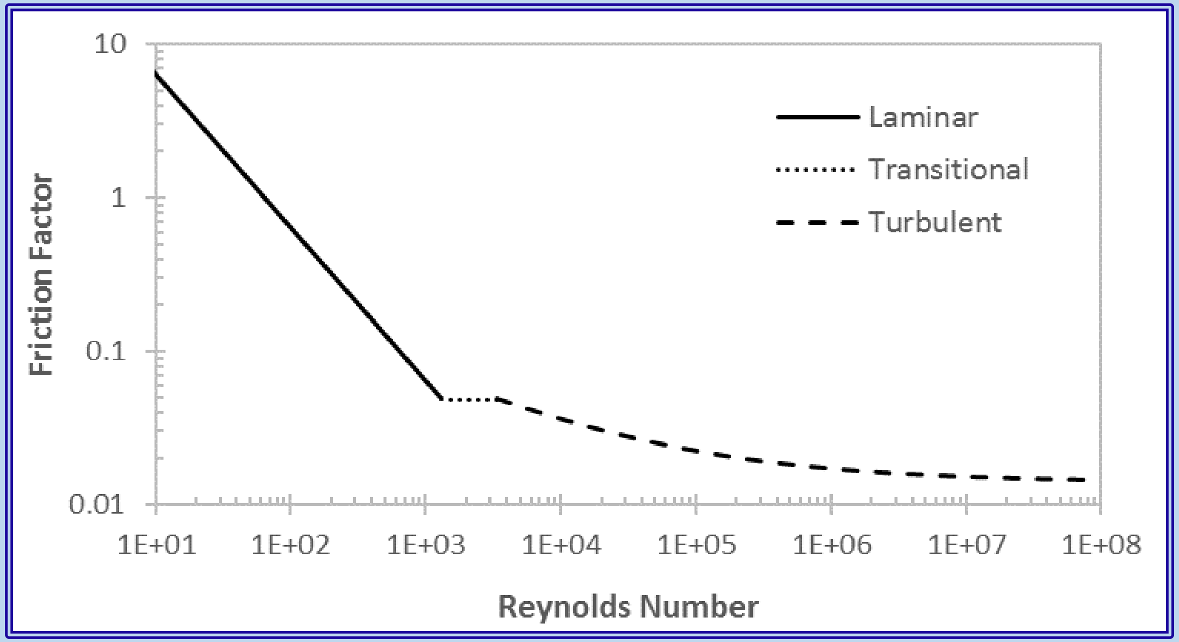
Thermohydraulic Modeling

Example: Friction Factor – Tube-side

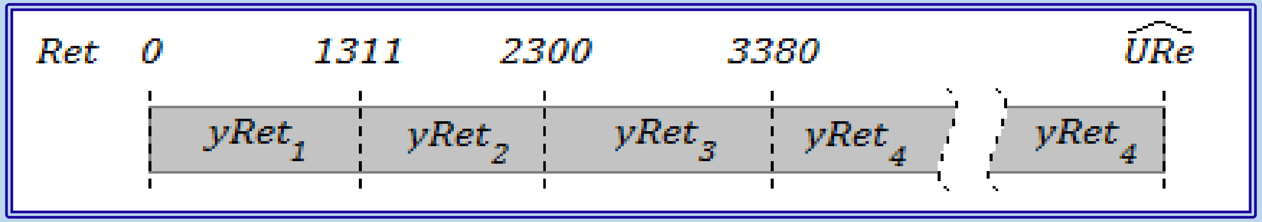
$$ft^{lam} = \frac{64}{Ret} \quad \text{for } Ret \leq 1311$$

$$ft^{tran} = 0.0488 \quad \text{for } 1311 < Ret \leq 3380$$

$$ft^{turb} = 0.014 + \frac{1.056}{Ret^{0,42}} \quad \text{para } Ret > 3380$$



Thermohydraulic Modeling



$$Ret \leq 1311\, yRet_1 + 2300\, yRet_2 + 3380\, yRet_3 + \widehat{URe}\, yRet_4$$

$$Ret \geq 1311 yRet_2 + 2300 yRet_3 + 3380\, yRet_4 + \varepsilon$$

$$\sum_{sRet=1}^{sRetmax} yRet_{sRet} = 1$$

⇩

$$ft = ft^{lam} yRet_1 + ft^{tran}\,(yRet_2 + yRet_3) + ft^{turb} yRet_4$$

Heat Transfer

Coefficients

$$ht = \frac{Nut\ kt}{dti} \qquad ha = \frac{Nua\ ka}{dh}$$

$$U = \frac{1}{\frac{1}{ht}\frac{dte}{dti} + Rft\frac{dte}{dti} + \frac{dte \ln\left(\frac{dte}{dti}\right)}{2\widehat{ktube}} + Rfa + \frac{1}{ha}}$$

Rate

$$\hat{Q} = U A_{req} \widehat{\Delta Tlm}\, F$$

$$F = 1 + \sum_{sE=2}^{sEmax} \sum_{sE'=2}^{sEmax} \{yT_c[yPt_{sE}(\widehat{pF}_{h,sE} - 1) + yPa_{sE'}(\widehat{pF}_{c,sE'} - 1)] + yT_h[yPt_{sE}(\widehat{pF}_{c,sE} - 1) + yPa_{sE'}(\widehat{pF}_{h,sE'} - 1)]\}$$

Area

$$A \geq \left(1 + \frac{\hat{A}_{exc}}{100}\right) A_{req} \qquad A = \pi\ dte\ Lu\ NB\ NPt\ NPa$$

Velocity and Pressure Drop Bounds

$$vt \geq \widehat{vt}_{min}$$

$$vt \leq \widehat{vt}_{max}$$

$$va \geq \widehat{va}_{min}$$

$$va \leq \widehat{va}_{max}$$

$$\Delta Pt \leq \widehat{\Delta P}_{c_{disp}} yT_c + \widehat{\Delta P}_{h_{disp}} yT_h$$

$$\Delta Pa \leq \widehat{\Delta P}_{c_{disp}} yT_h + \widehat{\Delta P}_{h_{disp}} yT_c$$

EVOLUTION OF THE PROPOSED MODEL

RAW MINLP

Mixed Integer Nonlinear Programming

MATHEMATICAL TRANSFORMATIONS

LINEAR-BINARY MILP

Mixed Integer Nonlinear Programming

- *Non-linearity restricted to continuous variables;*
- *Better resolution for Outer Approximation (OA).*

MILP

Mixed Integer Linear Programming

- *Linear constraints and objective function;*
- *Global optimality guaranteed.*

Solvers and Computer

MINLP 1

MINLP 2

⇨ *SBB*

MILP

⇨ *CPLEX*

Computer ⇨ *CPU Intel Core i7-6700 (16GB RAM)*

MINLP – Initial Estimates Required

Variable	Initial Estimate
Inner tube diameter selection	$yd_3 = 1$
Stream allocation	$yT_c = 1$
Range of Reynolds identification	$yRet_4 = yRea_4 = 1$
Range of Nusselt identification	$yNut_2 = yNua_2 = 1$

Variable	Initial Estimate
Length of one unit	$Lh = \frac{Lh_{lo} + Lh_{up}}{2}$
Reynolds number*	$Rex = \frac{Rex_{lo} + Rex_{up}}{2}$
Nusselt number*	$Nux = \frac{Nux_{lo} + Nux_{up}}{2}$
Seider & Tate Nusselt number*	$Nux^{S\&T} = \frac{Nux_{lo}^{S\&T} + Nux_{up}^{S\&T}}{2}$

Problem Dimension

Parameter	Number of discrete options
$\widehat{pNB}_{sB}$	6
$\widehat{pNE}_{sE}$	8
$\widehat{pL}_{sL}$	2
$\widehat{pdte}_{sd}$	4
$\widehat{pDte}_{sD}$	4

Problem Formulation	N° of constraints	N° of variables	Processing Time (s)
MINLP 1	106	135	12,5
MINLP 2	282	191	15,2
MILP	869.337	205.926	604,3

RESULTS

- Successful validation with literature results in terms of modelling.
- Improved results compared to commonly applied trial and verification procedures (up to 20% area reduction)
- Flexibility between different flow regimes and structural flexibility in response to imposed bounds successfully achieved.
- Both MINLP approaches achieve locally optimum results, sometimes with a difference of 50% between both results. No pattern for either of them achieving better solutions than the other was found.
- Due to the increase in the number of variables and constraints in the Linear-Binary MINLP approach its processing time is a bit larger than for the Raw MINLP approach.
- The MINLP solutions can also be used as a initial estimate for BARON, a global solver, and can reduce BARON computation effort significantly (up to 90%).
- The MILP approach guarantees global optimality and is not dependent on initial estimates, however it requires a much greater processing time, and can show memory limitation issues depending on the number of discrete variables considered, which are being further explored.